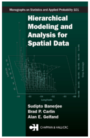
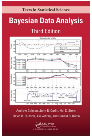
References

I have added the following reference to the course website:

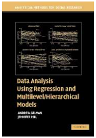


S. Banerjee, B. P. Carlin, and A. E. Gelfand. *Hierarchical Modeling and Analysis for Spatial Data*. Chapman and Hall 2003.

These are great references for Bayesian and hierarchical modeling.



A. Gelman *et al. Bayesian Data Analysis*. Third Edition. Chapman and Hall 2013.



A. Gelman and J. Hill. *Data Analysis Using Regression and Multilevel/Hierarchical Models*. Cambridge University Press 2006.

Lecture 11
Model Checking and Diagnostics

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Stats 253

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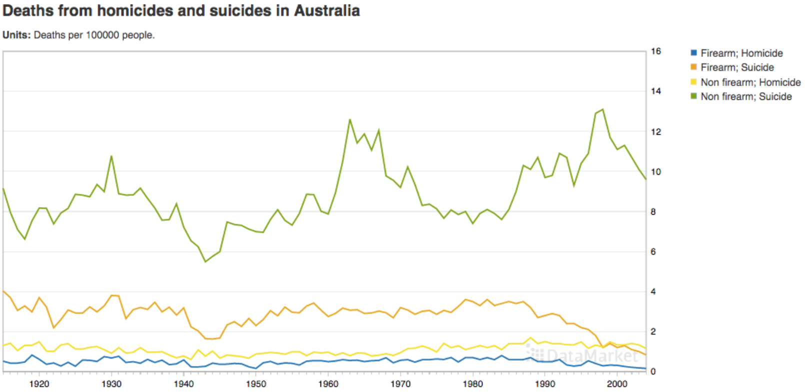
Australia's Gun Buyback Program

In 1996, Australia instituted a gun buyback program in an attempt to curb firearm deaths.

The numbers of deaths per year, categorized by type of death (homicide or suicide) and instrument (firearm or no firearm), are shown below.



Australia's Gun Buyback Program

Let's develop a hierarchical model that will help us determine if the gun buyback program helped reduce the number of deaths from firearms in Australia.

- Let y_{ijt} be the number of deaths in year t. $i = 0, 1$ depending on whether or not it was a homicide. $j = 0, 1$ depending on whether or not it was due to a firearm.
- What should the distribution of y_{ijt} be?
- How do we model correlation across time periods?
- How should we incorporate the gun buyback program into the model?

Issues

- The distribution of $\boldsymbol{\theta}^{(k)}$ converges to $p(\boldsymbol{\theta}|\mathbf{y})$ if we run MCMC long enough, but how long is long?
- We want multiple samples from our posterior. Can we just take $\boldsymbol{\theta}^{(k)}, \boldsymbol{\theta}^{(k+1)}, \ldots$?

Convergence Diagnostics

Gelman-Rubin diagnostic:

- Run m chains starting at an overdispersed set of points.

  For each parameter θ, compute between- and within-chain sum of squares after k iterations:

$$B = \frac{k}{m-1} \sum_{j=1}^{m} (\overline{\theta_j^{(\cdot)}} - \overline{\theta_\cdot^{(\cdot)}})^2$$

$$W = \frac{1}{m} \sum_{j=1}^{m} \frac{1}{k-1} \sum_{i=1}^{k} (\theta_j^{(i)} - \overline{\theta_j^{(\cdot)}})^2$$

Convergence Diagnostics

Gelman-Rubin diagnostic:

- The posterior variance can be estimated as

$$\widehat{\mathrm{Var}}[\theta|\mathbf{y}] = \frac{n-1}{n}W + \frac{1}{n}B,$$

  which generally overestimates the posterior variance, but is unbiased if we have converged to the true distribution.
- On the other hand, W underestimates the variance in general, but is unbiased as $k \to \infty$.
- The Gelman-Rubin statistic is $\hat{R} = \sqrt{\frac{\widehat{\mathrm{Var}}[\theta|\mathbf{y}]}{W}}$. This decreases to 1 as $k \to \infty$.
- We declare the chain as having converged if $\hat{R} \approx 1$ for **all parameters in the model**.
- Then, we typically discard the first half of each chain.

How many samples do I (effectively) have?

- After running the Gelman-Rubin diagnostic, we keep $\boldsymbol{\theta}^{(k/2)}, \boldsymbol{\theta}^{(k/2+1)}, \ldots$. How do we use these values?
- The marginal distribution of each $\boldsymbol{\theta}^{(i)}$ is (approximately) $p(\boldsymbol{\theta}|\mathbf{y})$, but they are not independent!
- For estimating the posterior mean, no problem:

$$\mathrm{E}\left[\frac{1}{k/2}\sum_{i=\frac{k}{2}+1}^{k}\boldsymbol{\theta}^{(i)} \mid \mathbf{y}\right] = \mathrm{E}[\boldsymbol{\theta}|\mathbf{y}].$$

- For estimating the posterior variance, there are some issues:

$$\mathrm{E}\left[\frac{1}{k/2-1}\sum_{i=\frac{k}{2}+1}^{k}(\boldsymbol{\theta}^{(i)} - \overline{\boldsymbol{\theta}})^2 \mid \mathbf{y}\right] \neq \mathrm{Var}[\boldsymbol{\theta}|\mathbf{y}].$$

- It is typical to **thin** the sequence by keeping only every ℓth draw: $\boldsymbol{\theta}^{(k/2)}, \boldsymbol{\theta}^{(k/2+\ell)}, \boldsymbol{\theta}^{(k/2+2\ell)}, \ldots$.

1. A Time Series Example

2. Diagnostics for MCMC

3. What’s Next

What's Next

- I would like to have at least one more lecture, on a topic of your choosing. I will send out a survey early next week.
- We will take Quiz 3 (on Bayesian modeling) during that lecture.
- Homework 3 will be posted early next week.
- Project proposals will be due Monday of the following week (to give you time to work on Homework 2, which is due Friday).
- I would like to have project presentations on Wednesday, August 5.